Reply to "Comment on 'Deterministic equations of motion and phase ordering dynamics'"

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We reply to the Comment by Kockelkoren and Chaté [Phys. Rev. E **65** 058101 (2002)].

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In the Comment by Kockelkoren and Chaté (KC) [1], it is addressed that the dynamic exponents z and λ of phase-ordering dynamics in the ϕ^4 theory with Hamiltonian equations of motion are $z=2$ and $\lambda=5/4$ rather than $z=2.6(1)$ and $\lambda=0.46(1)$ as reported in Ref. [2]. The data used by KC are up to $t=3000$ or 4000 for the domain size, and about $t=1000$ for other observables. The data for parameter set A reported in Ref. [1] are up to $t=640$, and we also have data for $t=1280$ available for comparison.

Kockelkoren and Chaté tend to believe that the logarithmic plot is misleading and the plot of $L(t)$ vs $t^{1/z}$ is more transparent. In Figs. 1(a) and 1(b) in the KC Comment, they *assume* $z=2$ and display the figures for its reasonability. We observe that the plot of $L(t)$ vs $t^{1/z}$ is not very sensitive to z. These two figures alone do not mean so much to us. One needs more analysis. For parameter set A, we plot the inverse autocorrelation $1/A(t)$ vs t^a with $a=0.625$ and 0.46 in Fig. 1. For comparison, we have enlarged the time scale for $a=0.46$ to $3t^a$. Extra calculations confirm that finite size and finite Δt effects are negligibly small here. The value $a=0.625$ corresponds to $z=2$ while $a=0.46$ implies $z=2.6$. Even though this kind of plot is in general not very sensitive to a, in this case the behavior is still quite different for these two values of a. The value $a=0.46$ obviously gives a better straight line. If we do not compare these two curves, the curve for $a=0.625$ looks similar to that in Fig. 1(a) in the KC Comment. It is misleading. We do not have complete data for the domain size $L(t)$ and similar analysis cannot be carried out. From a figure of $L(t)$ vs $t^{1/z}$ with $z=2.6$ for parameter set A offered by KC (not shown in the Comment), it seems that z might be between 2 and 2.6.

Figures 2(c) and 2(d) in the KC Comment do not contradict with our results and show that the exponent λ for the ϕ^4 theory is close to $\lambda=5/4$.

In literature, the logarithmic plot is a typical tool used to explore the power-law behavior of physical observables. Locale slopes of a curve may reveal the trend of corrections to the power law. In Figs. 1(c) and 1(d) and Figs. 2(a) and 2(b) in the KC Comment, such plots are displayed for the domain size $L(t)$ and the autocorrelation $A(t)$, even though the fluctuations of the local slopes are large. Kockelkoren and Chaté conclude that these plots are misleading, compared with Figs. 1(a) and 1(b). We cannot agree with this. In Fig. 2(a) (parameter set A), the slope of $A(t)$ looks relatively stable in late times. Our measurements of the local slopes with larger windows confirm this. The final value $\lambda/z=0.46(1)$ is a reasonable estimate. It is hard to believe that the asymptotic slope of the curve is given by the dashed line in the figure, i.e., $\lambda/z=0.625$. In Fig. 2(b) (parameter set B), the local slope seems to show an increasing trend at late times although the fluctuations are large. We will not be surprised if someone concludes a final value close to $\lambda/z=0.46$ with careful analysis of the data (e.g., with larger windows for the local slopes).

Figures 1(c) (parameter set A) and 1(d) (parameter set B) in the KC Comment are subtle. In general, the local slopes for set B change more dramatically than those for set A. But it looks like it slows down after $t=1000$. However, the behavior of set A is more or less similar. At least, it is difficult to conclude that the slopes for both set A and B converge to $1/z=0.5$. A value between 0.4 and 0.5 may be possible.

The data for the second moment are rough and we will not discuss it here.

In Sec. II of the KC Comment corrections to scaling are addressed. First, we think that erroneous conclusions in one system are not necessarily duplicated in another system. Second, both the dynamic exponent $z=2$ and the values for the correction exponents are also an assumption there and are not *extracted* in certain ways. As pointed out above, the plot of $L(t)$ vs $t^{1/z}$ is not so sensitive to z. Assuming $z=2$, both curves for set A and B are not too different from a straight line. With two more parameters K_1 and K_2 in Eq. (4), it is not surprising that one can observe a "good" fit.

Concerning phase-ordering dynamics of model C, it is not

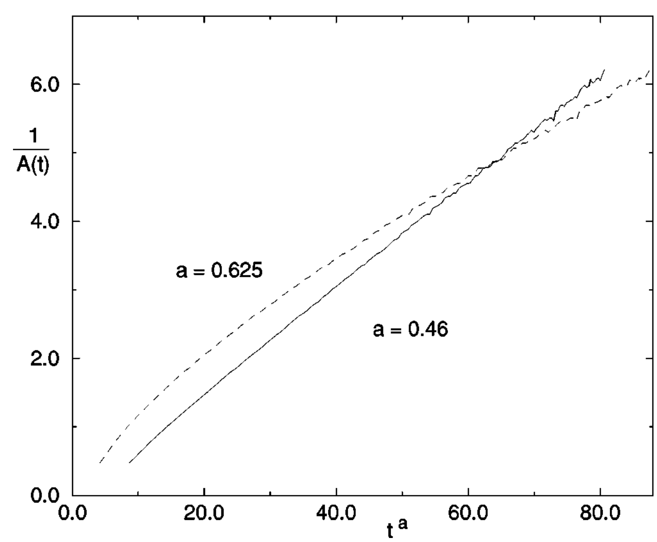

FIG. 1. The inverse autocorrelation for parameter set A with a lattice size $L=256$ and $\Delta t=0.01$.

unique, as is also pointed out in the KC Comment. If in one case it is model A-like and in another case model B-like, why can it not be that in the third case it is neither model A-like nor model B-like.

In conclusion, the KC Comment raises an interesting and important question. But their conclusion that $z=2$ and $\lambda=5/4$ is not sufficiently convincing. We cannot agree that the logarithmic plot is misleading but the plot of $L(t)$ vs $t^{1/z}$ is more transparent. For the data available, the exponents are effectively about $z=2.5$ and $\lambda/z=0.46$ for parameter set A and about $z=2$ and $\lambda=5/4$ for parameter set B. Whether and how the exponents for set A and B may converge still needs further investigation.

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[1] X. Kockelkoren and X. Chaté, Phys. Rev. E **65**, 058101 (2002).

[2] B. Zheng, Phys. Rev. E **61**, 153 (2000).